STATISTICAL CHARACTERISTICS OF THE PRESSURE FLUCTUATION IN A HYDRAULIC JUMP

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ABSTRACT: The author examines experimental data on the correlation in an essentially inhomogeneous, statistically stationary pressure fluctuation field at the bottom of a turbulent flow in the region of a hydraulic jump. Certain data relating to other statistical characteristics of the field in question (in particular, the one-dimensional distribution laws) may be found in [1].

The pressure p(q) at a rigid wall bounding a turbulent flow is a random function of the coordinates x and y of points on the surface of the wall and time t. The information about this random function required to solve practical problems is contained in its probability characteristics, the most widely used of which is the correlation function

$$R(x,\ x+\Delta x,\ y,\ y+\Delta y,\ t,\ t+\tau) = \langle p^0(\mathbf{q})\cdot p^0(\mathbf{q}+\Delta\mathbf{q})\rangle.$$

Here, q is a vector with components x, y, and t, the zero superscript indicates that the random function has been centered, while the angle brackets denote the operation of finding the mathematical expectation.

In the general case the correlation function of a three-dimensional field has six arguments. Important simplifications follow from homogeneity, when the probability characteristics are the same for any value of q and the correlation function depends only on three arguments: Δx, Δy and τ. In this paper the field is assumed homogeneous with respect to t (stationary) and with respect to y, and the surface R(x, Δx, Δy, τ) is investigated.

The stationarity considerably simplifies the formulation of the experiment, since the ergodicity of most stationary functions makes it possible to replace averaging with respect to the ensemble by averaging with respect to time. This assumption is valid for steady-state flows. Homogeneity with respect to y applies only to the central part of flows whose depth is much less than their width. By contrast with the stationarity assumption, if this latter assumption is discarded, the effect on the investigation is quantitative rather than qualitative.

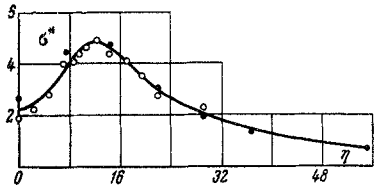


Fig. 1

The random function p(q) and its probability characteristics also depend on the nature of the phenomenon, the configuration of the flow boundaries, their rigidity, etc. The subject of investigation was an ideal hydraulic jump in a channel of rectangular cross section with a smooth horizontal bottom. The width of the flow was three times greater than the tailwater depth. The flow boundaries were absolutely rigid. The Froude number in the contracted section of the stream flowing from under a plane sluice gate with a sharp edge was 33; the Reynolds number of the averaged flow was $1.3\cdot10^5$. The resolving power of the measuring apparatus and the method of data analysis made it possible to obtain the statistical characteristics of fluctuations at frequencies from 0 to 50 Hz without systematic errors. The x axis was directed along the axis of the channel; the coordinate origin coincided with the contracted section of the stream. A more detailed description of the experimental conditions may be found in [1], where the geometric scale λ should be taken equal to two.

As distinct from the method described in [1], the ordinates of the oscillographic records (on punched tape in binary code) were read out by means of a facsimile system. The time quantization step was 0.0064 sec; the amplitude quantization ranged over 32 levels.

The fluctuation intensity is estimated by the quantity $\sigma = [R(x,0,0,0)]^{1/2}$. A graph of the function $\sigma^*(x) = (2g/v_1^2)\sigma(x)$ (where $v_1^2/2g$ is the velocity head in the contracted section) is presented in

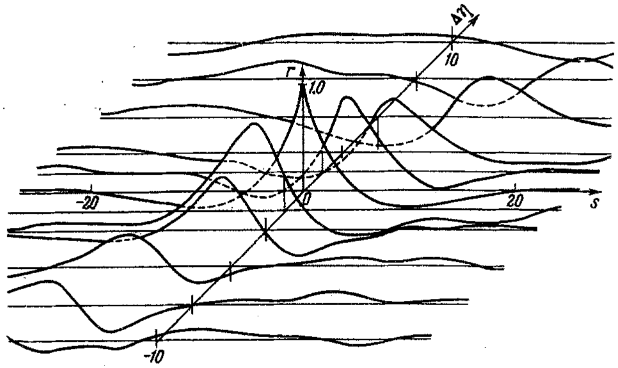


Fig. 2

Fig. 1. Along the axis of abscissas we have plotted the quantity $\eta = x/h_1$, where h_1 is the depth of the flow in the contracted section. In this figure the solid circles denote points obtained in previous investigations [1]. The fluctuation intensity is maximal at η = 12 (at a distance of roughly 1/3 of the length of the horizontal projection of the cylinder from the beginning of the jump). The coefficient of variation of the pressure in this zone reaches a value of 16%.

Figure 2 shows a series of sections of the normalized surface r(η, Δη, 0, s) by planes Δη = const at η = 10; here, $s = v_1\tau/h_1$ is the Strouhal number. The method of constructing these sections was as follows. Signals from two probes located along the axis of the channel at a distance Δx were simultaneously recorded on the oscillogram. The recording time T was not less than 30 secs. The data were analyzed on a digital computer using the algorithms

$$R(x,\Delta x,0,\tau) = \frac{1}{T-\tau}\int_0^{T-\tau} p^\circ(x,0,t)\,p^\circ(x+\Delta x,0,t+\tau)\,dt,$$

$$R(x,\Delta x,0,-\tau) = \frac{1}{T-\tau}\int_0^{T-\tau} p^\circ(x,0,t+\tau)\,p^\circ(x+\Delta x,0,t)\,dt.$$

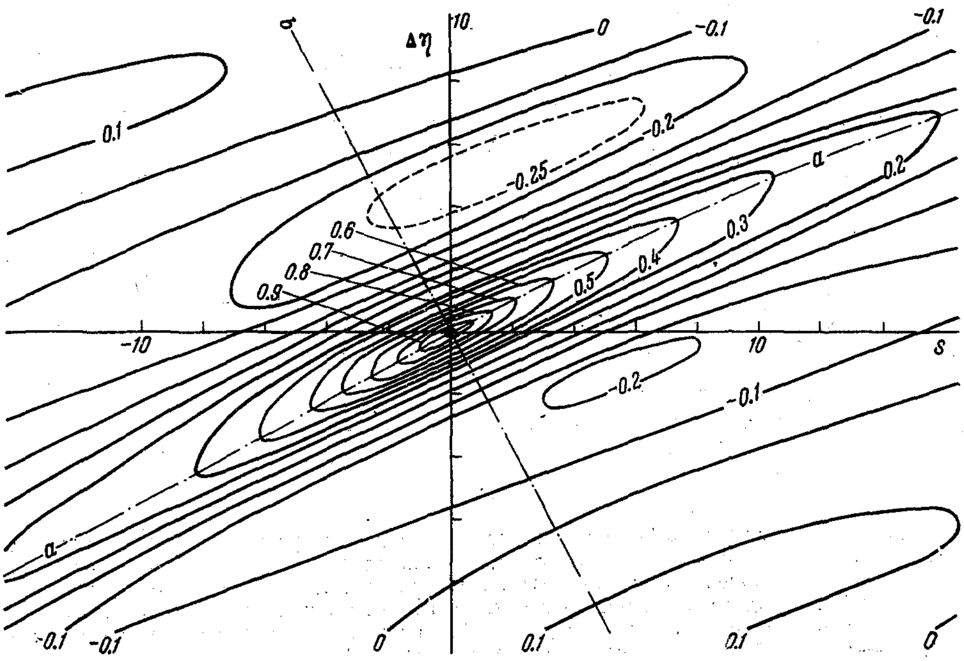
The normalization algorithm

$$r(x,\Delta x,0,\tau) = \frac{R(x,\Delta x,0,\tau)}{\sqrt{R(x,0,0,0)\,R(x+\Delta x,0,0,0)}}$$

is such that the normalized surface nowhere exceeds unity (in absolute value).

The experimental isolines of this surface then constructed were smoothed by the method of least squares, the approximation being based on a third-degree polynomial. The result is presented in Fig. 3. We note that the line *a*–*a* in this figure is not a straight line.

Figure 4 shows a series of characteristic sections of this surface, the abscissas being as follows: for curve *a* the dimensionless distance



b
Δη
10
0
-0.1
-0.1
0.1
-0.2
-0.25
a
0.2
0.6
0.7
0.8
0.9
0.3
0.5
0.4
-10
10
s
-0.2
-0.1
a
-0.1
-0.1
0
0.1
0.1
0

Fig. 3

$\Delta\eta$, for curve b the distance along the line a-a, for curve c the distance along the line b-b, and for curve d the dimensionless time s.

Curve a represents the correlations along the length of the jump in the vicinity of the point with $\eta = 10$. Its asymmetry is a consequence of the essential inhomogeneity of the field with respect to the x coordinate.

Section b by a cylindrical surface with directrix along the line a-a and perpendicular generatrix is of interest. The field in question is characterized not only by the curvature of the line a-a but also by a rapid decrease in the ordinates of the curve with distance from the coordinate origin.

Curve c is obtained in the section of the surface by a plane passing through the coordinate origin at right angles to the line a-a. Finally, curve d in this figure is the autocorrelation function at the point $\eta = 10$. It is symmetrical and was obtained by averaging 10 experimental curves, each of which was computed with the usual accuracy.

The sections most easily determined by experiment are those of the type $R(x, 0, 0, \tau)$, where x plays the part of parameter. In a homogeneous field one such section (the autocorrelation function at an arbitrary point) is sufficient to construct the entire surface. It is natural to try to use such sections to establish the complete correlation

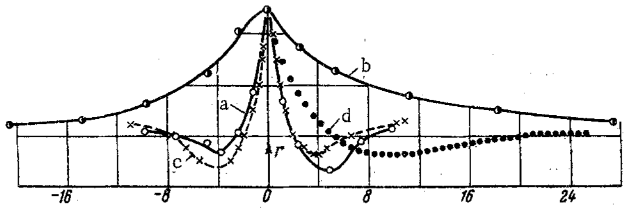


Fig. 4

picture in the case of an inhomogeneous field also [2]. Therefore the detailed study of these sections is of great importance. Our experimental curves are presented in Fig. 5, where along the ordinate axis we have plotted the square of the quantity

$$K = \frac{2g\sqrt{R(x, 0, 0, \tau)}}{v_1^2}.$$

Even the shape of the autocorrelation functions varies with x. However, their tendency to broaden (narrowing of the frequency spectrum) with distance from the beginning of the jump can be traced quite distinctly.

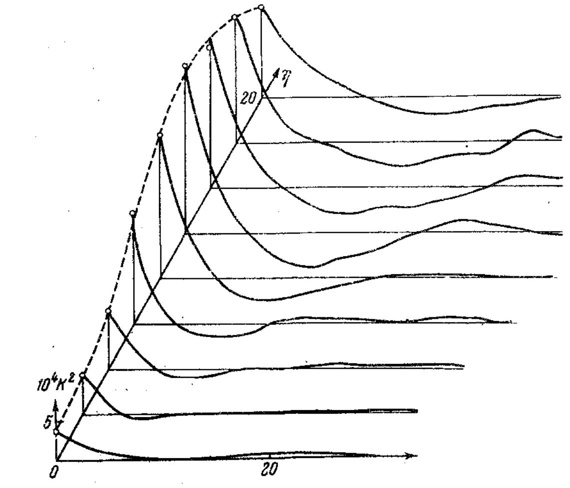


Fig. 5

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